

ADVANCED GCE
MATHEMATICS
Mechanics 2

4729

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

- Scientific or graphical calculator

Friday 18 June 2010
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 A particle is projected horizontally with a speed of 7 m s^{-1} from a point 10 m above horizontal ground. The particle moves freely under gravity. Calculate the speed and direction of motion of the particle at the instant it hits the ground. [6]

2 (i)

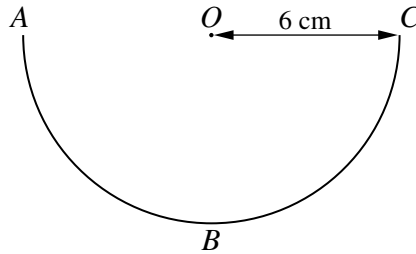


Fig. 1

A uniform piece of wire, ABC , forms a semicircular arc of radius 6 cm . O is the mid-point of AC (see Fig. 1). Show that the distance from O to the centre of mass of the wire is 3.82 cm , correct to 3 significant figures. [2]

(ii)

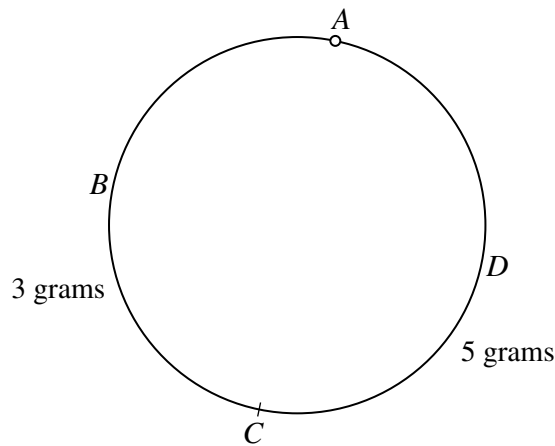


Fig. 2

Two semicircular pieces of wire, ABC and ADC , are joined together at their ends to form a circular hoop of radius 6 cm . The mass of ABC is 3 grams and the mass of ADC is 5 grams . The hoop is freely suspended from A (see Fig. 2). Calculate the angle which the diameter AC makes with the vertical, giving your answer correct to the nearest degree. [5]

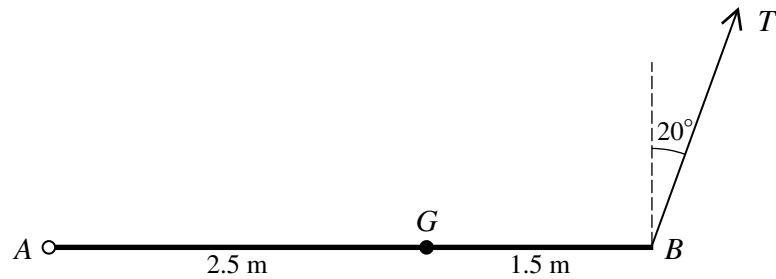
- 3 The maximum power produced by the engine of a small aeroplane of mass 2 tonnes is 128 kW . Air resistance opposes the motion directly and the lift force is perpendicular to the direction of motion. The magnitude of the air resistance is proportional to the square of the speed and the maximum steady speed in level flight is 80 m s^{-1} .

(i) Calculate the magnitude of the air resistance when the speed is 60 m s^{-1} . [5]

The aeroplane is climbing at a constant angle of 2° to the horizontal.

(ii) Find the maximum acceleration at an instant when the speed of the aeroplane is 60 m s^{-1} . [4]

4

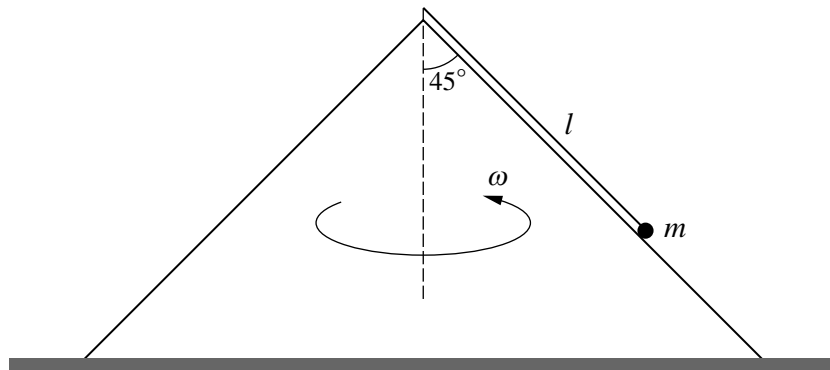


A non-uniform beam AB of length 4 m and mass 5 kg has its centre of mass at the point G of the beam where $AG = 2.5$ m. The beam is freely suspended from its end A and is held in a horizontal position by means of a wire attached to the end B . The wire makes an angle of 20° with the vertical and the tension is T N (see diagram).

(i) Calculate T . [3]

(ii) Calculate the magnitude and the direction of the force acting on the beam at A . [7]

5



One end of a light inextensible string of length l is attached to the vertex of a smooth cone of semi-vertical angle 45° . The cone is fixed to the ground with its axis vertical. The other end of the string is attached to a particle of mass m which rotates in a horizontal circle in contact with the outer surface of the cone. The angular speed of the particle is ω (see diagram). The tension in the string is T and the contact force between the cone and the particle is R .

(i) By resolving horizontally and vertically, find two equations involving T and R and hence show that $T = \frac{1}{2}m(\sqrt{2}g + l\omega^2)$. [6]

(ii) When the string has length 0.8 m, calculate the greatest value of ω for which the particle remains in contact with the cone. [4]

[Questions 6 and 7 are printed overleaf.]

- 6 A particle A of mass $2m$ is moving with speed u on a smooth horizontal surface when it collides with a stationary particle B of mass m . After the collision the speed of A is v , the speed of B is $3v$ and the particles move in the same direction.

(i) Find v in terms of u . [3]

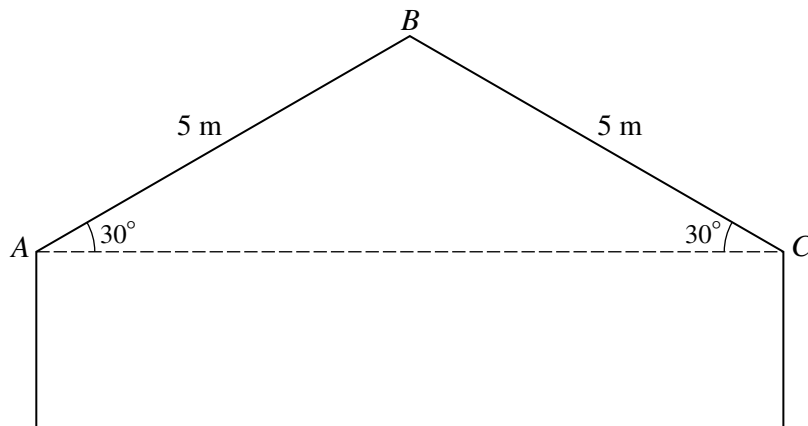
(ii) Show that the coefficient of restitution between A and B is $\frac{4}{5}$. [2]

B subsequently hits a vertical wall which is perpendicular to the direction of motion. As a result of the impact, B loses $\frac{3}{4}$ of its kinetic energy.

(iii) Show that the speed of B after hitting the wall is $\frac{3}{5}u$. [4]

(iv) B then hits A . Calculate the speeds of A and B , in terms of u , after this collision and state their directions of motion. [8]

7



A small ball of mass 0.2 kg is projected with speed 11 m s^{-1} up a line of greatest slope of a roof from a point A at the bottom of the roof. The ball remains in contact with the roof and moves up the line of greatest slope to the top of the roof at B . The roof is rough and the coefficient of friction is $\frac{1}{2}$. The distance AB is 5 m and AB is inclined at 30° to the horizontal (see diagram).

(i) Show that the speed of the ball when it reaches B is 5.44 m s^{-1} , correct to 2 decimal places. [6]

The ball leaves the roof at B and moves freely under gravity. The point C is at the lower edge of the roof. The distance BC is 5 m and BC is inclined at 30° to the horizontal.

(ii) Determine whether or not the ball hits the roof between B and C . [7]

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1	$v^2 = 2 \times 9.8 \times 10$ $v = 14 \text{ m s}^{-1}$ speed = $\sqrt{7^2 + 14^2}$ 15.7 or $7\sqrt{5} \text{ m s}^{-1}$ $\tan^{-1}(14/7)$ or $\tan^{-1}(7/14)$ 63.4° to the horizontal	M1 A1 M1 A1 M1 A1 6	Using $v^2 = u^2 + 2as$ with $u = 0$ Method to find speed using their “v” Method to find angle using their “v” 26.6° to vertical 6
2 (i)	$(6 \sin \Pi/2) \div (\Pi/2)$ 3.82	M1 A1 2	Use of correct formula AG
	(ii)	M1 A1 A1 M1 A1 5	Method to find centre of mass Attempt to find the required angle 7
3 (i)	$D = 128\,000/80 (= 1600)$ $k(80)^2 = 128\,000/80$ $k = 1/4$ $R = 900 \text{ N}$	B1 M1 A1 A1 B1 5	Driving force = resistance FT on their k ($R = 3600k$)
	(ii)	B1 B1 M1 A1 4	4 terms required 9
4 (i)	$4T \cos 20^\circ = 5 \times g \times 2.5$ $T = 32.6 \text{ N}$	M1 A1 A1 3	Using moments; allow sin/cos mix Allow with omission of g
	(ii)	M1 A1 M1 A1 M1 A1 A1 A1 7	allow sin/cos mix FT their T FT their T, but not from omission of g $X \neq 0, Y \neq 0$ or 31.2° to left of vertical 10

5 (i)	$T\cos 45^\circ + R\sin 45^\circ = mg$ $T\sin 45^\circ - R\cos 45^\circ = m\sin 45^\circ \omega^2$ $2T = \sqrt{2}mg + ml\omega^2$ $T = m/2(\sqrt{2}g + l\omega^2)$	*M1 A1 *M1 A1 Dep*M1 A1 6	3 terms 3 terms; $a = r\omega^2$ Method to eliminate R AG www
	(ii) $R = 0$ $2R = \sqrt{2}mg - ml\omega^2$ or $T\cos 45^\circ = mg$ or $T = ml\omega^2$ Solve to find ω $\omega = 4.16 \text{ rad s}^{-1}$	B1 B1 M1 A1 4	may be implied 10
6 (i)	$2mu = 2mv + 3mv$ $v = 2/5 u$	M1 A1 A1 3	Conservation of momentum Must be $v =$
	(ii) $e = (3v - v) / u$ $e = 4/5$	M1 A1 2	Using restitution AG
	(iii) Initial K.E. = $9mv^2 / 2 = 18mu^2 / 25$ Final K.E. = $9mv^2 / 8 = 9mu^2 / 50$ $\frac{1}{2} m (V)^2 = \text{Final K.E.}$ $V = 3u / 5$	B1 FT B1 FT M1 A1 4	FT on their v from (i) FT on their v from (i) AG
	(iv) $4mu / 5 - 3mu / 5 = 2mx + my$ $u / 5 = 2x + y$ $e = 4/5 = (y - x) / u$ $4u = 5y - 5x$ solving 2 relevant equations $x = -u/5$ $y = 3u/5$ $y = 3u/5$ away from wall (x) + towards wall (y)	M1 A1 FT M1 FT A1 M1 A1 A1 A1 8	Conservation of momentum FT on their v from (i); aef Using restitution FT on their v from (i); aef both 17

<p>7 (i)</p> <p>Or last 4 marks of (i)</p>	<p>$R = 0.2 \times 9.8 \times \cos 30^\circ (= 1.70)$ $F = 0.1 \times 9.8 \times \cos 30^\circ (= 0.849)$ FT</p> <p>$\frac{1}{2} \times 0.2 \times 11^2 - \frac{1}{2} \times 0.2 v^2 =$ $0.2 \times 9.8 \times 5 \sin 30 + 5 \times 0.849$ $v = 5.44 \text{ m s}^{-1}$</p> <p>$F + 0.2g \sin 30 = \pm 0.2a$ $a = \pm 9.1$ $v^2 = 11^2 + 2 \times a \times 5$ $v = 5.44 \text{ m s}^{-1}$</p>	<p>B1 B1 M1 A1 A1 A1 6 M1 A1 M1 A1</p>	<p>FT on their R, but not $R = 0.2g$ Use of conservation of energy</p> <p>AG</p> <p>Use of N2L, 3 terms</p> <p>Complete method to find v</p>
<p>(ii)</p> <p>Or first 5 marks of (ii)</p>	<p>$t = 5 \cos 30^\circ / 5.44 \cos 30^\circ$ $t = 0.919 \text{ s}$ $u = 5.44 \sin 30^\circ (= 2.72)$ $s = 2.72 \times 0.919 - 4.9 \times 0.919^2$ $s = -1.6$ (or better) Ht drop to C = $5 \sin 30^\circ = 2.5 \text{ m}$ Ball does not hit the roof</p> <p>$y = x \tan \theta - gx^2 \sec^2 \theta / 2V^2$ substitute values $V = 5.44 \quad \theta = 30^\circ \quad x = 5 \cos 30^\circ$ $y = 2.5 - 9.8 \times 25 \times 3 / 4 \times 4 / 3 / (2 \times 5.44^2)$ $y = -1.6$ (or better)</p>	<p>M1 A1 B1 M1 A1 B1 A1 7 B1 M1 A1 A1 A1</p>	<p>time to lateral position over C</p> <p>Ht dropped</p> <p>13</p> <p>all 3 correct</p>
<p>OR (ii)</p>	<p>$u = 5.44 \sin 30^\circ (= 2.72)$ $-2.5 = 5.44 \sin 30 t - 4.9 t^2$</p> <p>$t = 1.04$ $x = 5.44 \cos 30 \times 1.04 = 4.9$ (or better) Horizontal distance from B to C = $5 \cos 30 = 4.3$ (or better) Ball does not hit the roof</p>	<p>B1 M1 A1 A1 A1 B1 A1 7</p>	<p>aef time to position level with AC</p>
<p>OR (ii)</p>	<p>$y = x \tan \theta - gx^2 \sec^2 \theta / 2V^2$ substitute values $-2.5 = 0.577x - 0.221x^2$ Attempt to solve quadratic for x $x = 4.9$ (or better) Horizontal distance from B to C = $5 \cos 30 = 4.3$ (or better) Ball does not hit the roof</p>	<p>B1 M1 A1 M1 A1 B1 A1 7</p>	<p>aef</p>
<p>OR (ii)</p>	<p>$u = 5.44 \sin 30^\circ = 2.72$ $-2.5 = 5.44 \sin 30 t - 4.9 t^2$</p> <p>$t = 1.0$ (or better) $T = 5 \cos 30^\circ / 5.44 \cos 30^\circ$ $T = 0.92$ (or better) Ball does not hit the roof</p>	<p>B1 M1 A1 A1 M1 A1 A1 7</p>	<p>aef time to position level with AC time to lateral position over C</p>

OR (ii)	Attempt at equation of trajectory $y = 0.577x - 0.221x^2$ $y = -0.577x$ Solving their quadratic and linear equations to get at least x or y $x = 5.2$ (or better) or $y = -3.0$ (or better) Horizontal distance from B to C = $5\cos 30 = 4.3$ (or better) Or Ht drop to C = $5\sin 30^\circ = 2.5$ Ball does not hit the roof	M1 A1 B1 M1 A1 B1 A1 7	Equation of BC Must be the one needed for comparison
OR (ii)	Attempt at equation of trajectory $y = 0.577x - 0.221x^2$ $y = -0.577x$ Solving their quadratic and linear equations $x = 5.2$ (or better) and $y = -3.0$ (or better) Distance = 6.0 (or better) Ball does not hit the roof	M1 A1 B1 M1 A1 B1 A1 7	Distance from B to point of intersection